

On the Capacity of Mobile Ad hoc Networks with Delay Constraints^{*}

Cristina Comaniciu and H. Vincent Poor
Department of Electrical Engineering
Princeton University
Princeton, NJ 08544
e-mail: ccomanic, poor@ee.princeton.edu

ABSTRACT

Previous work on ad hoc network capacity has focused primarily on source-destination throughput requirements for different models and transmission scenarios, with an emphasis on delay tolerant applications. In such problems, network capacity enhancement is achieved as a tradeoff with transmission delay. In this paper, the capacity of ad hoc networks supporting delay sensitive traffic is studied. To enhance the network capacity, advanced signal processing techniques such as multiuser detection, which can be implemented adaptively and blindly, are relied upon. For system capacity derivation, results from random graph theory are combined with an asymptotic physical layer analysis for three different network scenarios employing matched filters, decorrelators and minimum-mean-square-error receivers. Insight into the network performance for finite systems is also provided by means of simulations. Both analysis and simulations show a significant network capacity gain for ad hoc networks employing multiuser detectors, compared with those using matched filter receivers, as well as very good performance even under tight delay and transmission power requirements.

1 Introduction

A mobile ad hoc network consist of a group of mobile nodes which spontaneously form temporary networks without the aid of a fixed infrastructure or centralized management. The communication between any two nodes can either be direct or relayed through other nodes (if the direct transmission causes too much interference in the network). The ad hoc networks research literature has been traditionally focused on routing and medium access control and only recently there has been an increased interest in characterizing the capacity of such networks. We mention here a few landmark papers that analyze network capacity in terms of achievable

throughput under different system models and assumptions [6, 8, 10]. While in [10] the authors focus on fixed, finite networks and derive capacity regions under various predefined transmission protocols, and considering omniscient nodes, [8] and [6] discuss the asymptotic throughput performance for fixed and mobile networks, respectively. In [8], the authors study the capacity of a fixed ad hoc network in which the nodes' locations are fixed but randomly distributed. They prove that, as the number of nodes (N) per unit area increases, the achievable throughput between any randomly selected source-destination pair is on the order of $O(1/\sqrt{N})$. In contrast to this somewhat pessimistic result, [6] shows that exploiting mobility can result in a form of multiuser diversity and can improve the system capacity. The authors of [6] propose a two-hop transmission strategy in which the traffic is first randomly spread (first hop) across as many relay nodes as possible and it is delivered (second hop) as soon as the relaying nodes are close to the destination. The disadvantage of this scheme is that it involves large delays and therefore it is not suitable for delay sensitive traffic. A capacity increase with mobility has also been noticed in [7], in which the capacity is empirically determined for a different network model that exploits spatial diversity.

In this paper, we study the capacity of large mobile ad hoc networks carrying delay sensitive traffic. Because of tight delay requirements, we cannot take advantage of mobility as in [6]. To improve the capacity we rely on advanced signal processing techniques such as multiuser detection, which can be implemented adaptively and blindly (e.g. [13]).

We analyze the network for a given stationary distribution of the mobile nodes' locations with constraints on the maximum number of hops between any arbitrary source-destination pair. Using arguments similar to those in [8] we show that limiting the maximum number of hops for any given transmission also improves the source-destination throughput by limiting the additional transmissions for the relayed traffic. On the other hand, reducing the number of hops has a negative impact on the capacity by increasing

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the interference level. Thus, for delay sensitive traffic, the network capacity is interference limited and multiuser receivers can significantly improve the performance.

For the network capacity derivation, we use results from random graph theory [4]. A random graph is characterized by the number N of nodes and the probability p of maintaining a link between two arbitrary nodes. Using the results proved in [3] we can obtain a constraint on the probability of maintaining a link such that the graph's diameter is D with high probability as the number of nodes goes to infinity. As these results are asymptotic in nature we also validate them through simulations for finite values of N . The graph diameter represents the longest shortest path between any two nodes, and consequently, is the maximum number of hops required for transmission between any given pair of nodes. The probability p characterizes the physical layer and is defined as the probability that the signal-to-interference ratio can be maintained above the desired target. We compute p for different scenarios (Code-division multiple-access (CDMA) with random spreading codes and matched filter, minimum-mean-square-error (MMSE) and decorrelating receivers) using an asymptotic analysis (both the number of nodes and the spreading gain are taken to infinity while maintaining their ratio constant) [11].

Random graph theory results have been previously used for ad hoc networks in the context of call admission control [5]. In [5], the authors emphasized the importance of network connectivity and network admission control criteria were proposed. No specific physical layer analysis was provided. In this paper, we study the interplay between network layer quality of service (QoS) requirements (delay and throughput) modeled using random graph theory, and physical layer QoS requirements (signal-to-interference ratio target) with an emphasis on capacity gains obtained through signal processing.

2 System Model

We consider an ad hoc network consisting of N mobile nodes, having a uniform stationary distribution over a square area, of dimension $D^* \times D^*$. The multiaccess scheme is synchronous direct-sequence CDMA (DS-SS) and three types of receivers are considered: matched filter (MF), decorrelator, and linear minimum mean squared error. All nodes use independent, randomly generated and normalized spreading sequences of length L . For simplicity, we assume that all nodes transmit with the same power, P_t , and we define the signal-to-noise ratio (SNR), as the ratio between the transmitted power and the noise power: $SNR = P_t/\sigma^2$. The analysis considers the free space propagation model for which the received power is given as:

$$P_r = P_t^* G_t G_r \frac{\lambda^2}{(4\pi d)^2} = P_t \frac{\lambda^2}{d^2} = P_t h, \quad (1)$$

where P_t represents the transmitted power which incorporates also the transmitting and receiving antenna gains and the constant $1/(4\pi)^2$, λ is the wavelength, d is the distance between the transmitter and the receiver, and h is the link gain.

The traffic can be directly transmitted between any two nodes, or it can be relayed through intermediate nodes. It is assumed that at each time slot the packets travel one hop, so that the end-to-end delay can be measured as the number of hops required for a route to be completed. The QoS requirements for the ad hoc network are the bit error rate (mapped into a signal-to-interference ratio (SIR) requirement), the average source-destination throughput (T_{S-D}), and the transmission delay. Both the throughput and the delay are influenced by the maximum number of hops allowed for a connection, and consequently by the network diameter D . Using arguments similar to those in [8], a simplified computation shows that, if the number of hops for a transmission is D , each node generates $D\lambda(N)$ traffic for other nodes ($\lambda(N)$ represents the traffic generation rate for a given node). Thus, the total traffic in the network has to meet the stability condition $D\lambda(N)N \leq W/L$, where W is the system bandwidth. This implies that the maximum average source-destination throughput that can be supported by the network is

$$T_{S-D} = \frac{W}{LD}. \quad (2)$$

Thus, lower network diameter constraints will ensure lower transmission delays and higher source-destination throughputs for the network.

In terms of SIR requirements, a connection can be established between two nodes if the SIR is greater than or equal to the target SIR γ . The obtained SIR for a particular link is random due to the randomness of the nodes' positions. To compute the probability of a connection between any two nodes we rely on results developed in [9] on the distribution of distances between any two nodes, when the nodes' locations are uniformly distributed in a rectangular area. In [9] the authors have obtained an exact distribution for the distances between any two users that are uniformly distributed in a rectangular area. They also have proved that the obtained cumulative distribution function (CDF) (and probability density function (pdf), respectively), can be well approximated by the CDF obtained considering an alternate model, in which the nodes are distributed according to a Gaussian distribution having the standard deviation $\sigma_1 = D^*/k$, with $k = 3.5$. The CDF for the Gaussian model is given by

$$F_d(y = k\sigma_1 x) = 1 - \exp\left(-\frac{k^2}{4}x^2\right). \quad (3)$$

Equivalently, (3) can be expressed as

$$F_d(y) = 1 - \exp\left(-\frac{k^2}{4D^{*2}}y^2\right). \quad (4)$$

The similarity between the two models is illustrated in Fig. 1 for an example with $D^* = 20$. For simplicity, we use the expression in (4) throughout the analysis, while the simulations rely on the actual uniform distribution over the square area.

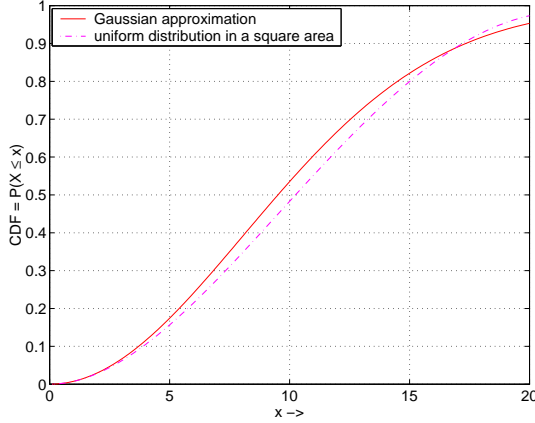


Figure 1. Gaussian approximation model: CDF

We denote by $\delta_m = \lambda$ the minimum distance for reception and by $\delta_M = \sqrt{2}D^*$ the maximum distance between two nodes (nodes uniformly distributed in a square area). Hence, the link gain h takes values in the interval $[\lambda^2/\delta_m^2, \lambda^2/\delta_M^2]$ with high probability (e.g. according to the Gaussian model $P(d \leq \delta_m) \approx 8.5033 \times 10^{-4}$, and $P(d \geq \delta_M) \approx 0.0022$ for $\lambda = 0.1$ m and $D^* = 6$). As a consequence, the CDF for the link gain can be expressed as follows:

$$F_H(h) = 1 - F_d\left(\lambda^2/\sqrt{h}\right) = \exp\left(-\frac{C}{h}\right), \quad h > 0, \quad (5)$$

where $C = \frac{k^2}{4D^{*2}}\lambda^2$.

Taking the derivative of (5) we obtain the probability density function for the link gain:

$$f_H(h) = \frac{C}{h^2} \exp\left(-\frac{C}{h}\right), \quad h \geq 0. \quad (6)$$

Using (6) the mean link gain can be easily computed to be

$$E_H \approx C [E_1(\delta_m^2 C) - E_1(\delta_M^2 C)], \quad (7)$$

where $E_1(x) = \int_x^\infty \frac{1}{t} \exp(-t) dt$ is the exponential integral.

We define the network capacity to be the maximum number of nodes that can be supported such that both the SIR

constraints and the delay constraints can be met for any arbitrary source-destination pair of nodes. We map the delay constraints into a maximum network diameter constraint D . To characterize the network capacity we use results from random graph theory, where the ad hoc network represents a random graph $G(N, p)$ with N nodes and the probability of a link between any two nodes being equal to p . The probability p is given by the physical layer such that the SIR constraints are met.

We characterize the ad hoc network asymptotic capacity for the case for which the number of nodes and the spreading gain go to infinity, while their ratio is fixed.

3 Asymptotic Capacity

To characterize the ad hoc network capacity we rely on results from random graph theory [3], which determine the network diameter for a given link probability value p , when the number of nodes goes to infinity. We summarize the main results needed (from [3]) in the following theorem:

Theorem 1 [3] *Let c be a positive constant, $D = D(N) \geq 2$ a natural number, and define $p = p(N, c, D)$, $0 < p < 1$, by*

$$p^D N^{D-1} = \log\left(\frac{N^2}{c}\right). \quad (8)$$

Suppose that $(pN)/(\log N)^3 \rightarrow \infty$. Then in $G(N, p)$ we have

$$\lim_{N \rightarrow \infty} P(\text{diam} G = D) = e^{-c/2}, \text{ and} \\ \lim_{N \rightarrow \infty} P(\text{diam} G = D + 1) = 1 - e^{-c/2}$$

In Fig. 2 we illustrate the theoretical link probability requirements for an ad hoc network with a finite number of nodes for given network diameter requirements, derived according to Theorem 1. The constant c was selected such that the probability that the network has diameter D is 0.99. In Section 4 we will discuss the theorem's applicability for such finite networks.

From Fig. 2 we notice that network diameter guarantees can be translated into physical layer guarantees using a link probability requirement. The link probability is affected by the level of interference and thus it will be very sensitive to the choice of receiver.

We derive the asymptotic capacity regions for three types of receivers: matched filter, decorrelator and linear MMSE, under specific constraints on the link probability p .

Matched Filter

The SIR condition for an arbitrary user i using a matched filter receiver in a network with random, normalized spreading sequences can be expressed as

$$SIR_i = \frac{h_i}{SNR^{-1} + \frac{1}{L} \sum_{j=1, j \neq i}^N h_j} \geq \gamma, \quad (9)$$

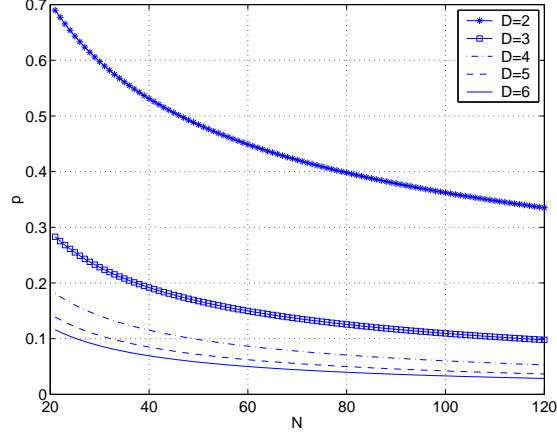


Figure 2. Link probability requirement for different delay constraints

when all users transmit with equal powers.

Denoting by α the fixed ratio N/L and letting the number of nodes and the spreading gain go to infinity, by using the law of large numbers [14], it follows that $\frac{1}{L} \sum_{j=1, j \neq i}^N h_j \rightarrow \alpha E_H$, with E_H computed as in (7). The network diameter guarantees require a link probability value equal to p . This translates into a physical layer condition

$$P(h \geq \gamma SNR^{-1} + \alpha \gamma E_H) = P(h \geq Thr_{MF}) = p. \quad (10)$$

Using the notation $Thr_{MF} = \gamma SNR^{-1} + \alpha \gamma E_H$, the network diameter condition renders an SNR condition

$$\gamma SNR^{-1} + \alpha \gamma E_H = Thr_{MF} \Rightarrow SNR = \frac{\gamma}{Thr_{MF} - \alpha \gamma E_H}, \quad (11)$$

where Thr_{MF} can be derived using (5) as follows

$$p = 1 - F_H(Thr_{MF}) = 1 - \exp\left(-C \frac{1}{Thr_{MF}}\right); \quad (12)$$

$$Thr_{MF} = \frac{C}{\log\left(\frac{1}{1-p}\right)}. \quad (13)$$

Equation (11) implies that a positive power solution exists if and only if

$$\alpha_{MF} < \frac{Thr_{MF}}{\gamma E_H} = \frac{\frac{C}{\log\left(\frac{1}{1-p}\right)}}{\gamma E_H}. \quad (14)$$

For ad hoc networks, it is most likely that the mobile nodes are energy limited and so we assume that a maximum power transmission limit \bar{P}_t is imposed. Denoting $SNR_c = \bar{P}_t/\sigma^2$, the ad hoc network capacity region becomes

$$\alpha_{MF} \leq \frac{Thr_{MF}}{\gamma E_H} - \frac{1}{E_H SNR_c} = \frac{\frac{C}{\log\left(\frac{1}{1-p}\right)}}{\gamma E_H} - \frac{1}{E_H SNR_c}. \quad (15)$$

Decorrelator

According to results presented in [11], the SIR of an arbitrary user in an asymptotically large network using decorrelating receivers can be expressed as:

$$SIR_d = \begin{cases} \frac{P_t h(1-\alpha)}{\sigma^2}, & \alpha < 1 \\ 0, & \alpha \geq 1. \end{cases} \quad (16)$$

Thus, if no power constraints are imposed the network capacity region is

$$\alpha_d < 1. \quad (17)$$

If power constraints are imposed, and a maximum SNR_c is allowed, the physical layer constraint can be expressed as:

$$P\left(h \geq \frac{\gamma}{SNR_c(1-\alpha)}\right) = p. \quad (18)$$

Similarly to the matched filter case, we can define Thr_d , which is computed using the same formula (13), and thus

$$SNR = \frac{\gamma}{Thr_d(1-\alpha)} \leq SNR_c. \quad (19)$$

Consequently, the asymptotic capacity region for a network using decorrelating receivers and having transmission power constraints is given as

$$\alpha_d \leq 1 - \frac{\gamma}{Thr_d SNR_c} = 1 - \frac{\gamma}{\frac{C}{\log\left(\frac{1}{1-p}\right)} SNR_c}. \quad (20)$$

MMSE Detector

To derive the asymptotic ad hoc network capacity region for the MMSE detector we first express the SIR for an arbitrary user i in a large network using MMSE receivers, as in [11], for equal transmitted powers for all users:

$$SIR_i = \frac{h_i}{SNR^{-1} + \frac{1}{L} \sum_{j=1, j \neq i}^N \frac{h_i h_j}{h_i + h_j \gamma}}. \quad (21)$$

Denoting $\alpha = N/L$, as the number of nodes and the spreading gain go to infinity, we can apply the law of large numbers to yield

$$\frac{1}{L} \sum_{j=1, j \neq i}^N \frac{h_i h_j}{h_i + h_j \gamma} = \alpha \frac{1}{N} \sum_{j=1, j \neq i}^N \frac{h_i h_j}{h_i + h_j \gamma} \rightarrow \alpha E[H/h_i],$$

where we used the notation $E[H/h_i]$ to denote the normalized conditional average interference (normalized to the number of users per dimension). It can be shown that $E[H/h_i]$ can be expressed as:

$$\begin{aligned} E[H/h_i] &= \\ &= C \exp\left(\frac{C\gamma}{h_i}\right) \left[E_1\left(\delta_m^2 C + \frac{C\gamma}{h_i}\right) - E_1\left(\delta_M^2 C + \frac{C\gamma}{h_i}\right) \right]. \end{aligned} \quad (22)$$

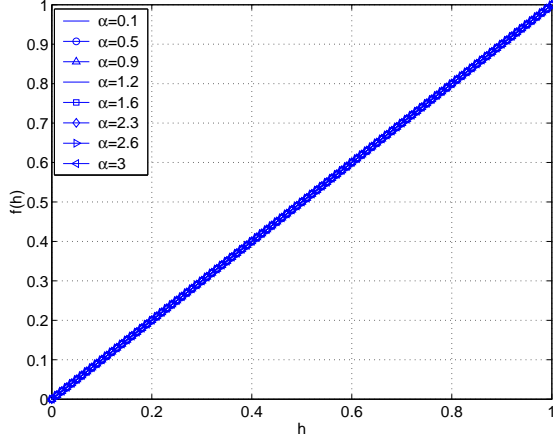


Figure 3. SIR condition monotonicity

Thus, the link probability constraint becomes

$$P(h \geq \gamma SNR^{-1} + \alpha \gamma E[H/h]) = p. \quad (23)$$

We define the function $f(h) = h - \gamma SNR^{-1} - \alpha \gamma E[H/h]$ and we plot it in Fig. 3. We observe that $f(h)$ is a monotonically increasing function of h for the region of interest, and thus, we can express the condition (23) as

$$P(h \geq Thr_{MMSE}) = p. \quad (24)$$

Equation (24) has the same solution as in the previously analyzed cases, and the physical layer constraint becomes:

$$SNR = \frac{\gamma}{Thr_{MMSE} - \alpha \gamma E[H/h = Thr_{MMSE}]}. \quad (25)$$

A positive transmit power solution exists if and only if

$$\alpha_{MMSE} < \frac{Thr_{MMSE}}{\gamma E[H/h = Thr_{MMSE}]}; \quad (26)$$

or equivalently,

$$\alpha_{MMSE} < \frac{\frac{C}{\log\left(\frac{1}{1-p}\right)}}{\gamma C \left(\frac{1}{1-p}\right)^\gamma \left[E_1\left(\delta_m^2 C + \gamma \log\left(\frac{1}{1-p}\right)\right) - E_1\left(\delta_M^2 C + \gamma \log\left(\frac{1}{1-p}\right)\right)\right]}. \quad (27)$$

If power constraints are imposed, the capacity region becomes

$$\alpha_{MMSE} \leq \frac{Thr_{MMSE}}{\gamma E[H/h = Thr_{MMSE}]} - \frac{1}{E[H/h = Thr_{MMSE}] SNR_c}. \quad (28)$$

Figure 4 illustrates the physical layer capacity as a function of the link probability constraint for the three receivers

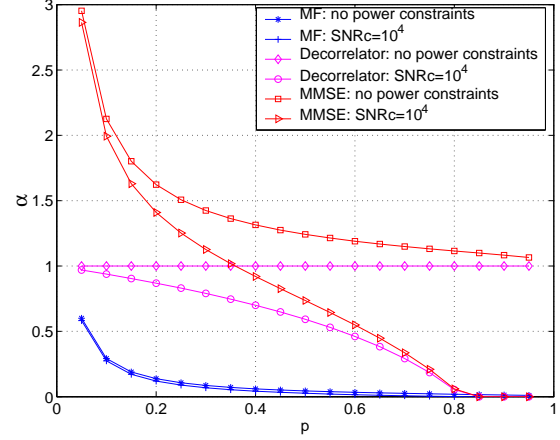


Figure 4. Physical layer capacity for given link probability constraint

considered and with or without power constraints. For the power-constrained case, a maximum transmission power of $\bar{P}_t = 10^4 \sigma^2$ is considered for this example.

The ad hoc network capacity is determined by combining (8) with the above derived capacity equations for the physical layer, with $\alpha = N/L$. We note that the link probability p expressed as a characteristic of the physical layer is a constant for all analyzed scenarios for the asymptotic case (both the number of users and the spreading gain are driven to infinity, while maintaining their ratio constant). This ensures that $(pN)/(\log N)^3 \rightarrow \infty$, and the assumptions of Theorem 1 hold.

Thus, for different average source-destination throughput requirements, the maximum number of users in the ad hoc network can be determined as a function of delay constraints (expressed as network diameter constraints). Figure 5 illustrates an example for the ad hoc network capacity for the three types of receivers when $P_t \leq \bar{P}_t = 10^4 \sigma^2$, for two different values of the spreading gain.

From Figs. 4 and 5, we notice that there is a significant capacity advantage if multiuser receivers are used, and conversely, for given capacity requirements, substantial power savings can be achieved by networks using multiuser receivers. As expected, the MMSE receiver performs the best due to its property of maximizing the SIR. For higher transmission rates and lower delay requirements, using the matched filter is not feasible.

An important observation is that the actual achieved average source-destination throughput decreases proportionally with the network diameter D ($T_{S-D} = W/(LD)$), as discussed in the previous section.

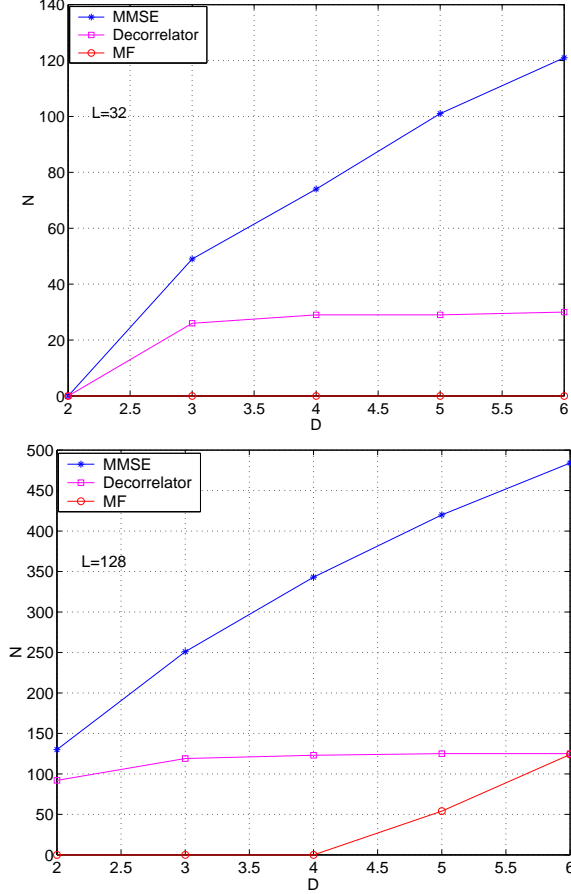


Figure 5. Asymptotic capacity for different delay constraints: $SNR_c = 10^4$

4 Finite Networks Simulations

The capacity results obtained in the previous section are asymptotic in nature, thus requiring validation through simulations for practical finite networks. All numerical results in this section are obtained using $D^* = 6$, $\lambda = 0.1$ m and $\gamma = 5$. Since we showed in the previous section that the network using matched filters performs poorly compared with a system using multiuser detectors, the emphasis is on networks using multiuser receivers, and the results are only validated for the matched filter case. All the experiments consider unlimited power transmission for the MMSE case, and maximum power constraints for the decorrelator, $\bar{P}_t = 10^4 \sigma^2$ (the case with unlimited transmission power is trivial: $\alpha < 1$).

The conducted experiments consist of selecting a finite (variable) number of nodes and randomly generating their locations uniformly across a square area. Then, the link gains, and consequently the achieved SIRs are computed for all pairs of nodes, using Eqs. (9), (16), and (21), re-

spectively. We note that the simulations do not consider the SIR formulas' accuracy for finite systems, as this issue has already been studied in [12], where it was shown that the standard deviation for the achieved SIR goes to zero as $1/\sqrt{N}$.

If the computed SIR is greater than or equal to the target SIR, the link is feasible. The adjacency matrix is then constructed, and based on it, the network diameter is determined. The computation of the network diameter uses Dijkstra's algorithm [2], as a Matlab function from the Bayes Net Toolbox package [1]. The experiment is repeated 100 times and the probabilities associated with a range of network diameters are determined. An infinite network diameter means that the network is disconnected.

We also determine the probability of a feasible link p and we compare it with the theoretical, asymptotic results. Some simulation examples are presented in Tables 1 and 2. It can be seen that the physical layer capacity results, reflected in the achievable link probability p , are very close to the asymptotic ones, whereas the network performance is strongly affected. The physical layer capacity performance quickly approaches that of the asymptotic system as the number of nodes grows.

On the other hand, the effect of the reduced number of nodes on the obtained network diameter is very strong. In Table 1 we can observe how the network diameter drifts to a higher value than the one expected, as the link probability decreases. For very small values of p , the obtained network diameter also begins to spread across multiple values. Similar behavior was observed for the decorrelator and for the matched filter case. We also noticed that there are break-point probabilities for which the network diameter changes, and they appear to be invariant to the receiver type and to the number of nodes for the range of nodes considered for the simulation. This is best illustrated in Fig. 6, for a large range for the number of users (nodes) in the network.

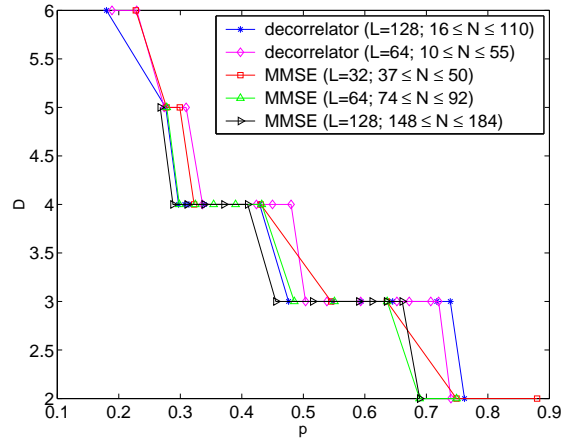


Figure 6. Network diameter: simulations

Table 1. Simulation results: MMSE; (A)= asymptotic analysis results, (S)=simulation results.

N/L	$p(A)/p(S)$	D(A)/D(S)
38/32	0.6056/0.7491	D=2
39/32	0.5415/0.4886	D=2
42/32	0.4024/0.433	D=3
45/32	0.3137/0.3260	D=3
46/32	0.2913/0.2983	D=3
48/32	0.2537/0.2590	D=3
57/32	0.1546/0.1584	D=3
78/64	0.5415/0.5490	D=2
74/64	0.6814/0.7482	D=2

Table 2. Simulation results: MF; (A)= asymptotic analysis results, (S)=simulation results.

N/L	$p(A)/p(S)$	D(A)/D(S)
44/1024	0.51/0.61	D=2
31/256	0.22/0.31	D=3
144/512	0.10/0.11	D=3

In Fig. 6 we notice that the probability break points remain approximately the same for all analyzed cases, with smaller networks favoring slightly higher points than larger networks. Thus, we can design a practical network using the diameter/probability dependence determined experimentally in Fig. 6. From simulations, we determine that a network diameter of $D = 2$ can be obtained for a link probability $p \geq 0.7$ and a network diameter of $D = 3$ can be obtained for $p \geq 0.55$ for a large range for the number of nodes.

Finally, using the link probability values experimentally determined, the ad hoc network capacity for practical finite systems can be determined for given delay (network diameter) constraints. Figure 7 illustrates the network capacity for a network diameter constraint of $D = 2$. Figure 7 shows the number of users per dimension that can be supported in an ad hoc network for a given delay constraint, as a function of the maximum transmission power requirement, $SNR_c = \bar{P}_t/\sigma^2$.

It can be seen that, using multiuser receivers, almost cellular capacity (obtained for the case with multiuser receivers) can be obtained even for very stringent delay ($D = 2$) and power requirements (transmission power $\bar{P}_t = 10^5 \sigma^2$).

5 Conclusions

In this paper we have analyzed the asymptotic capacity for delay sensitive traffic in ad hoc networks. While previous results focus on enhancing the network capacity at the expense of increased transmission delay, our approach is to exploit advanced signal processing techniques, such as multiuser detection, to enhance capacity when tight delay constraints are enforced. We have analyzed three different

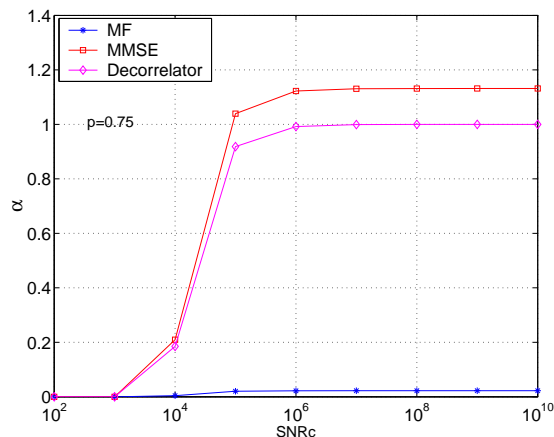


Figure 7. Ad hoc network capacity for delay sensitive traffic: $D=2$

network scenarios for a DS-CDMA air interface in which the users have matched filters, decorrelating or MMSE receivers. We have combined physical layer requirements (signal to interference ratio) with network layer QoS constraints (transmission delay). The maximum network transmission delay has been expressed in terms of the maximum number of hops for any arbitrary selected source-destination pair of nodes. We then have characterized the network delay using results from random graph theory, related to the network diameter. Since all derivations in this paper are asymptotic in nature, simulation results have been presented for performance validation with finite systems. While experiments have revealed a very close match for the physical layer performance compared with the asymptotic system, the network performance is seen to be strongly degraded compared to the asymptotic case. It has been shown that an overdesign for the physical layer is required for a finite network, compared to the theoretical performance obtained for an asymptotic one. Based on these simulations, general trends for capacity have been observed, and have been shown to hold for a large range of network dimensions. As expected, the performance improves for both the physical and the network layer as the number of nodes in the network increases. Both analysis and simulations have shown significant network capacity gains for ad hoc networks employing multiuser detectors, compared with those using matched filters, as well as a very good performance even under tight delay and power constraints.

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